

NCERT Solutions Class 11 Maths Chapter 12 Introduction to Three Dimensional Geometry

Question 1:

A point is on the x -axis. What are its y -coordinate and z -coordinates?

Solution:

If a point is on the x -axis, then the coordinates of y and z are 0.

So, the point is $(x, 0, 0)$.

Question 2:

A point is in the XZ -plane. What can you say about its y -coordinate?

Solution:

If a point is in XZ -plane, then its y -coordinate is 0.

Question 3:

Name the octants in which the following points lie:

$(1, 2, 3), (4, -2, 3), (4, -2, -5), (4, 2, -5), (-4, 2, -5), (-4, 2, 5), (-3, -1, 6), (2, -4, -7)$.

Solution:

Here is the table which represents the octants:

Octants	I	II	III	IV	V	VI	VII	VIII
x	+	−	−	+	+	−	−	+
y	+	+	−	−	+	+	−	−
z	+	+	+	+	−	−	−	−

- (i) $(1, 2, 3)$
Here x is positive, y is positive, and z is positive.
So, it lies in I octant.
- (ii) $(4, -2, 3)$
Here x is positive, y is negative, and z is positive.
So, it lies in IV octant.
- (iii) $(4, -2, -5)$
Here x is positive, y is negative, and z is negative.
So, it lies in VIII octant.

- (iv) $(4, 2, -5)$
Here x is positive, y is positive, and z is negative.
So, it lies in V octant.
- (v) $(-4, 2, -5)$
Here x is negative, y is positive, and z is negative.
So, it lies in VI octant.
- (vi) $(-4, 2, 5)$
Here x is negative, y is positive, and z is positive.
So, it lies in II octant.
- (vii) $(-3, -1, 6)$
Here x is negative, y is negative, and z is positive.
So, it lies in III octant.
- (viii) $(2, -4, -7)$
Here x is positive, y is negative, and z is negative.
So, it lies in VIII octant.

Question 4:

Fill in the blanks:

- (i) The x -axis and y -axis taken together determine a plane known as XY plane.
- (ii) The coordinates of points in the XY-plane are of the form $(x, y, 0)$.
- (iii) Coordinate planes divide the space into eight octants.

EXERCISE 12.2

Question 1:

Find the distance between the following pairs of points:

- (i) $(2, 3, 5)$ and $(4, 3, 1)$
- (ii) $(-3, 7, 2)$ and $(2, 4, -1)$
- (iii) $(-1, 3, -4)$ and $(1, -3, 4)$
- (iv) $(2, -1, 3)$ and $(-2, 1, 3)$

Solution:

- (i) $(2, 3, 5)$ and $(4, 3, 1)$

Let P be $(2, 3, 5)$ and Q be $(4, 3, 1)$

By using the formula,

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

So here,

$$x_1 = 2, y_1 = 3, z_1 = 5$$

$$x_2 = 4, y_2 = 3, z_2 = 1$$

$$\begin{aligned} PQ &= \sqrt{(4-2)^2 + (3-3)^2 + (1-5)^2} \\ &= \sqrt{2^2 + 0^2 + (-4)^2} \\ &= \sqrt{4+16} \\ &= \sqrt{20} \\ &= 2\sqrt{5} \end{aligned}$$

Therefore, the required distance is $2\sqrt{5}$ units.

- (ii) $(-3, 7, 2)$ and $(2, 4, -1)$

Let P be $(-3, 7, 2)$ and Q be $(2, 4, -1)$

By using the formula,

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

So here,

$$x_1 = -3, y_1 = 7, z_1 = 2$$

$$x_2 = 2, y_2 = 4, z_2 = -1$$

$$\begin{aligned}
 PQ &= \sqrt{(2 - (-3))^2 + (4 - 7)^2 + (-1 - 2)^2} \\
 &= \sqrt{5^2 + (-3)^2 + (-3)^2} \\
 &= \sqrt{25 + 9 + 9} \\
 &= \sqrt{43}
 \end{aligned}$$

Therefore, the required distance is $\sqrt{43}$ units.

(iii) $(2, -1, 3)$ and $(-2, 1, 3)$

Let P be $(-1, 3, -4)$ and Q be $(1, -3, 4)$

By using the formula,

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

So here,

$$x_1 = -1, y_1 = 3, z_1 = -4$$

$$x_2 = 1, y_2 = -3, z_2 = 4$$

$$\begin{aligned}
 PQ &= \sqrt{(1 - (-1))^2 + (-3 - 3)^2 + (4 - (-4))^2} \\
 &= \sqrt{2^2 + (-6)^2 + 8^2} \\
 &= \sqrt{4 + 36 + 64} \\
 &= \sqrt{104} \\
 &= 2\sqrt{26}
 \end{aligned}$$

Therefore, the required distance is $2\sqrt{26}$ units.

(iv) $(2, -1, 3)$ and $(-2, 1, 3)$

Let P be $(2, -1, 3)$ and Q be $(-2, 1, 3)$

By using the formula,

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

So here,

$$x_1 = 2, y_1 = -1, z_1 = 3$$

$$x_2 = -2, y_2 = 1, z_2 = 3$$

$$\begin{aligned}
 PQ &= \sqrt{(-2-2)^2 + (1-(-1))^2 + (3-3)^2} \\
 &= \sqrt{(-4)^2 + 2^2 + 0^2} \\
 &= \sqrt{16+4} \\
 &= \sqrt{20} \\
 &= 2\sqrt{5}
 \end{aligned}$$

Therefore, the required distance is $2\sqrt{5}$ units.

Question 2:

Show that the points $(-2, 3, 5)$, $(1, 2, 3)$ and $(7, 0, -1)$ are collinear.

Solution:

If three points are collinear, then they lie on a line.

Firstly, let us calculate distance between the 3 points i.e. PQ, QR and PR.

Calculating PQ:

$$P = (-2, 3, 5) \text{ and } Q = (1, 2, 3)$$

By using the formula,

$$\text{Distance } PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

So here,

$$x_1 = -2, y_1 = 3, z_1 = 5$$

$$x_2 = 1, y_2 = 2, z_2 = 3$$

$$\begin{aligned}
 PQ &= \sqrt{[1 - (-2)]^2 + (2 - 3)^2 + (3 - 5)^2} \\
 &= \sqrt{(3)^2 + (-1)^2 + (-2)^2} \\
 &= \sqrt{9 + 1 + 4} \\
 &= \sqrt{14}
 \end{aligned}$$

Calculating QR:

$$Q = (1, 2, 3) \text{ and } R = (7, 0, -1)$$

By using the formula,

$$\text{Distance } QR = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

So here,

$$x_1 = 1, y_1 = 2, z_1 = 3$$

$$x_2 = 7, y_2 = 0, z_2 = -1$$

$$\begin{aligned} QR &= \sqrt{(7-1)^2 + (0-2)^2 + (-1-3)^2} \\ &= \sqrt{6^2 + (-2)^2 + (-4)^2} \\ &= \sqrt{36 + 4 + 16} \\ &= \sqrt{56} \\ &= 2\sqrt{14} \end{aligned}$$

Calculating PR:

$$P = (-2, 3, 5) \text{ and } R = (7, 0, -1)$$

By using the formula,

$$\text{Distance } PR = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

So here,

$$x_1 = -2, y_1 = 3, z_1 = 5$$

$$x_2 = 7, y_2 = 0, z_2 = -1$$

$$\begin{aligned} PR &= \sqrt{(7 - (-2))^2 + (0 - 3)^2 + (-1 - 5)^2} \\ &= \sqrt{9^2 + (-3)^2 + (-6)^2} \\ &= \sqrt{81 + 9 + 36} \\ &= \sqrt{126} \\ &= 3\sqrt{14} \end{aligned}$$

$$\text{Thus, } PQ = \sqrt{14}, QR = 2\sqrt{14} \text{ and } PR = 3\sqrt{14}$$

So,

$$\begin{aligned} PQ + QR &= \sqrt{14} + 2\sqrt{14} \\ &= 3\sqrt{14} \\ &= PR \end{aligned}$$

Therefore, the points P, Q and R are collinear.

Question 3:

Verify the following:

- (i) $(0, 7, -10), (1, 6, -6)$ and $(4, 9, -6)$ are the vertices of an isosceles triangle.
- (ii) $(0, 7, 10), (-1, 6, 6)$ and $(-4, 9, 6)$ are the vertices of a right angled triangle.

(iii) $(-1, 2, 1), (1, -2, 5), (4, -7, 8)$ and $(2, -3, 4)$ are the vertices of a parallelogram.

Solution:

(i) $(0, 7, -10), (1, 6, -6)$ and $(4, 9, -6)$ are the vertices of an isosceles triangle.

Let us consider the points be $P(0, 7, -10), Q(1, 6, -6)$ and $R(4, 9, -6)$

If any 2 sides are equal, hence it will be an isosceles triangle.

So, firstly let us calculate the length of the sides.

Calculating PQ:

$P(0, 7, -10)$ and $Q(1, 6, -6)$

By using the formula,

$$\text{Distance } PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

So here,

$$x_1 = 0, y_1 = 7, z_1 = -10$$

$$x_2 = 1, y_2 = 6, z_2 = -6$$

$$\begin{aligned} PQ &= \sqrt{(1-0)^2 + (6-7)^2 + (-6-(-10))^2} \\ &= \sqrt{1^2 + (-1)^2 + 4^2} \\ &= \sqrt{1+1+16} \\ &= \sqrt{18} \end{aligned}$$

Calculating QR:

$Q(1, 6, -6)$ and $R(4, 9, -6)$

By using the formula,

$$\text{Distance } QR = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

So here,

$$x_1 = 1, y_1 = 6, z_1 = -6$$

$$x_2 = 4, y_2 = 9, z_2 = -6$$

$$\begin{aligned} QR &= \sqrt{(4-1)^2 + (9-6)^2 + (-6-(-6))^2} \\ &= \sqrt{3^2 + 3^2 + 0^2} \\ &= \sqrt{9+9} \\ &= \sqrt{18} \end{aligned}$$

$$\text{Here, } PQ = QR = \sqrt{18}$$

Since, two sides are equal, ΔPQR is an isosceles triangle.

Thus, $(0, 7, -10)$, $(1, 6, -6)$ and $(4, 9, -6)$ are the vertices of an isosceles triangle.

- (ii) $(0, 7, 10)$, $(-1, 6, 6)$ and $(-4, 9, 6)$ are the vertices of a right angled triangle.

Let the points be $P(0, 7, 10)$, $Q(-1, 6, 6)$ and $R(-4, 9, 6)$

Firstly, let us calculate the length of sides PQ, QR and PR.

Calculating PQ:

$P(0, 7, 10)$ and $Q(-1, 6, 6)$

By using the formula,

$$\text{Distance } PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

So here,

$$x_1 = 0, y_1 = 7, z_1 = 10$$

$$x_2 = -1, y_2 = 6, z_2 = 6$$

$$\begin{aligned} PQ &= \sqrt{(-1-0)^2 + (6-7)^2 + (6-10)^2} \\ &= \sqrt{(-1)^2 + (-1)^2 + (-4)^2} \\ &= \sqrt{1+1+16} \\ &= \sqrt{18} \end{aligned}$$

Calculating QR:

$Q(-1, 6, 6)$ and $R(-4, 9, 6)$

By using the formula,

$$\text{Distance } QR = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

So here,

$$x_1 = -1, y_1 = 6, z_1 = 6$$

$$x_2 = -4, y_2 = 9, z_2 = 6$$

$$\begin{aligned} QR &= \sqrt{(-4-(-1))^2 + (9-6)^2 + (6-6)^2} \\ &= \sqrt{(-3)^2 + 3^2 + 0^2} \\ &= \sqrt{9+9} \\ &= \sqrt{18} \end{aligned}$$

Calculating PR:

$P(0,7,10)$ and $R(-4,9,6)$

By using the formula,

$$\text{Distance } PR = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

So here,

$$x_1 = 0, y_1 = 7, z_1 = 10$$

$$x_2 = -4, y_2 = 9, z_2 = 6$$

$$\begin{aligned} PR &= \sqrt{(-4-0)^2 + (9-7)^2 + (6-10)^2} \\ &= \sqrt{(-4)^2 + 2^2 + (-4)^2} \\ &= \sqrt{16+4+16} \\ &= \sqrt{36} \end{aligned}$$

Now,

$$\begin{aligned} PQ^2 + QR^2 &= 18 + 18 \\ &= 36 \\ &= PR^2 \end{aligned}$$

By using Converse of Pythagoras theorem,

The given vertices P, Q and R are the vertices of a right-angled triangle at Q.

Thus, $(0,7,10), (-1,6,6)$ and $(-4,9,6)$ are the vertices of a right angled triangle.

(iii) $(-1,2,1), (1,-2,5), (4,-7,8)$ and $(2,-3,4)$ are the vertices of a parallelogram.

Let the points be $A(-1,2,1), B(1,-2,5), C(4,-7,8)$ and $D(2,-3,4)$

if pairs of opposite sides are equal then only ABCD can be a parallelogram.

i.e., $AB = CD$ and $BC = AD$.

Firstly, let us calculate the lengths of the sides

Calculating AB:

$A(-1,2,1)$ and $B(1,-2,5)$

By using the distance formula,

$$\text{Distance } AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

So here,

$$x_1 = -1, y_1 = 2, z_1 = 1$$

$$x_2 = 1, y_2 = -2, z_2 = 5$$

$$\begin{aligned}
 AB &= \sqrt{(1 - (-1))^2 + (-2 - 2)^2 + (5 - 1)^2} \\
 &= \sqrt{2^2 + (-4)^2 + 4^2} \\
 &= \sqrt{4 + 16 + 16} \\
 &= \sqrt{36} \\
 &= 6
 \end{aligned}$$

Calculating BC:

$B(1, -2, 5)$ and $C(4, -7, 8)$

By using the distance formula,

$$\text{Distance } BC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

So here,

$$x_1 = 1, y_1 = -2, z_1 = 5$$

$$x_2 = 4, y_2 = -7, z_2 = 8$$

$$\begin{aligned}
 BC &= \sqrt{(4 - 1)^2 + (-7 - (-2))^2 + (8 - 5)^2} \\
 &= \sqrt{3^2 + (-5)^2 + 3^2} \\
 &= \sqrt{9 + 25 + 9} \\
 &= \sqrt{43}
 \end{aligned}$$

Calculating CD:

$C(4, -7, 8)$ and $D(2, -3, 4)$

By using the distance formula,

$$\text{Distance } CD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

So here,

$$x_1 = 4, y_1 = -7, z_1 = 8$$

$$x_2 = 2, y_2 = -3, z_2 = 4$$

$$\begin{aligned}
 CD &= \sqrt{[(2 - 4)^2 + (-3 - (-7))^2 + (4 - 8)^2]} \\
 &= \sqrt{(-2)^2 + 4^2 + (-4)^2} \\
 &= \sqrt{4 + 16 + 16} \\
 &= \sqrt{36} \\
 &= 6
 \end{aligned}$$

Calculating DA:

$D(2, -3, 4)$ and $A(-1, 2, 1)$

By using the formula,

$$\text{Distance } DA = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

So here,

$$x_1 = 2, y_1 = -3, z_1 = 4$$

$$x_2 = -1, y_2 = 2, z_2 = 1$$

$$\begin{aligned} DA &= \sqrt{(-1-2)^2 + (2-(-3))^2 + (1-4)^2} \\ &= \sqrt{(-3)^2 + 5^2 + (-3)^2} \\ &= \sqrt{9+25+9} \\ &= \sqrt{43} \end{aligned}$$

Since, in quadrilateral ABCD both the pairs of opposite sides are equal i.e., $AB = CD$ and $BC = AD$, ABCD is a parallelogram.

Thus, $(-1, 2, 1), (1, -2, 5), (4, -7, 8)$ and $(2, -3, 4)$ are the vertices of a parallelogram.

Question 4:

Find the equation of the set of points which are equidistant from the points $(1, 2, 3)$ and $(3, 2, -1)$.

Solution:

Let $A(1, 2, 3)$ and $B(3, 2, -1)$

Let point P be (x, y, z)

Since it is given that point $P(x, y, z)$ is equidistant from the points $A(1, 2, 3)$ and $B(3, 2, -1)$ i.e., $PA = PB$

Firstly, let us calculate distances PA and PB

Calculating PA:

$P(x, y, z)$ and $A(1, 2, 3)$

By using the distance formula,

$$\text{Distance } PA = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

So here,

$$x_1 = x, y_1 = y, z_1 = z$$

$$x_2 = 1, y_2 = 2, z_2 = 3$$

$$PA = \sqrt{(1-x)^2 + (2-y)^2 + (3-z)^2}$$

Calculating PB:

$$P(x, y, z) \text{ and } B(3, 2, -1)$$

By using the distance formula,

$$\text{Distance } PB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

So here,

$$x_1 = x, y_1 = y, z_1 = z$$

$$x_2 = 3, y_2 = 2, z_2 = -1$$

$$PB = \sqrt{(3-x)^2 + (2-y)^2 + (-1-z)^2}$$

Since, $PA = PB$

On squaring both the sides, we get

$$PA^2 = PB^2$$

Therefore,

$$\begin{aligned} (1-x)^2 + (2-y)^2 + (3-z)^2 &= (3-x)^2 + (2-y)^2 + (-1-z)^2 \\ (1+x^2-2x) + (4+y^2-4y) + (9+z^2-6z) &= (9+x^2-6x) + (4+y^2-4y) + (1+z^2+2z) \\ -2x-4y-6z+14 &= -6x-4y+2z+14 \\ 4x-8z &= 0 \\ x-2z &= 0 \end{aligned}$$

Thus, the required equation is $x-2z=0$

Question 5:

Find the equation of the set of points P, the sum of whose distances from $A(4,0,0)$ and $B(-4,0,0)$ is equal to 10.

Solution:

$$\text{Let } A(4,0,0) \text{ and } B(-4,0,0)$$

Let the coordinates of point P be (x, y, z)

Calculating PA:

$$P(x, y, z) \text{ and } A(4, 0, 0)$$

By using the distance formula,

Distance $PA = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

So here,

$$x_1 = x, y_1 = y, z_1 = z$$

$$x_2 = 4, y_2 = 0, z_2 = 0$$

Distance $PA = \sqrt{(4 - x)^2 + (0 - y)^2 + (0 - z)^2}$

Calculating PB:

$$P(x, y, z) \text{ and } B(-4, 0, 0)$$

By using the distance formula,

Distance $PB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

So here,

$$x_1 = x, y_1 = y, z_1 = z$$

$$x_2 = -4, y_2 = 0, z_2 = 0$$

Distance $PB = \sqrt{(-4 - x)^2 + (0 - y)^2 + (0 - z)^2}$

Now it is given that

$$PA + PB = 10$$

$$PA = 10 - PB$$

On squaring both the sides, we get

$$PA^2 = (10 - PB)^2$$

$$PA^2 = 100 + PB^2 - 20PB$$

Therefore,

$$(4 - x)^2 + (0 - y)^2 + (0 - z)^2 = 100 + (-4 - x)^2 + (0 - y)^2 + (0 - z)^2 - 20PB$$

$$(16 + x^2 - 8x) + y^2 + z^2 = 100 + (16 + x^2 + 8x) + y^2 + z^2 - 20PB$$

$$20PB = 16x + 100$$

$$5PB = (4x + 25)$$

On squaring both the sides again, we get

$$25PB^2 = 16x^2 + 200x + 625$$

$$25[(-4 - x)^2 + (0 - y)^2 + (0 - z)^2] = 16x^2 + 200x + 625$$

$$25[x^2 + y^2 + z^2 + 8x + 16] = 16x^2 + 200x + 625$$

$$25x^2 + 25y^2 + 25z^2 + 200x + 400 = 16x^2 + 200x + 625$$

$$9x^2 + 25y^2 + 25z^2 - 225 = 0$$

Thus, the required equation is $9x^2 + 25y^2 + 25z^2 - 225 = 0$.

EXERCISE 12.3

Question 1:

Find the coordinates of the point which divides the line segment joining the points $(-2, 3, 5)$ and $(1, -4, 6)$ in the ratio (i) 2:3 internally, (ii) 2:3 externally.

Solution:

- (i) The coordinates of point R that divides the line segment joining the points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ internally in the ratio $m:n$ are $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right)$

Let $R(x, y, z)$ be the point that divides the line segment joining points $(-2, 3, 5)$ and $(1, -4, 6)$ internally in the ratio 2:3

Hence,

$$x = \frac{2(1) + 3(-2)}{2+3}, y = \frac{2(-4) + 3(3)}{2+3} \text{ and } z = \frac{2(6) + 3(5)}{2+3}$$

$$\text{i.e., } x = \frac{-4}{5}, y = \frac{1}{5} \text{ and } z = \frac{27}{5}$$

Thus, the coordinates of the required point are $\left(\frac{-4}{5}, \frac{1}{5}, \frac{27}{5}\right)$.

- (ii) The coordinates of point R that divides the line segment joining the points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ internally in the ratio $m:n$ are $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right)$

Let $R(x, y, z)$ be the point that divides the line segment joining points $(-2, 3, 5)$ and $(1, -4, 6)$ externally in the ratio 2:3

Hence,

$$x = \frac{2(1) - 3(-2)}{2-3}, y = \frac{2(-4) - 3(3)}{2-3} \text{ and } z = \frac{2(6) - 3(5)}{2-3}$$

$$\text{i.e., } x = -8, y = 17 \text{ and } z = 3$$

Thus, the coordinates of the required point are $(-8, 17, 3)$.

Question 2:

Given that $P(3, 2, -4)$, $Q(5, 4, -6)$ and $R(9, 8, -10)$ are collinear. Find the ratio in which Q divides PR.

Solution:

Let point $Q(5, 4, -6)$ divides the line segment joining points $P(3, 2, -4)$ and $R(9, 8, -10)$ in the ratio $k:1$.

Therefore, by section formula,

$$(5, 4, -6) = \left(\frac{k(9) + 3}{k + 1}, \frac{k(8) + 2}{k + 1}, \frac{k(-10) - 4}{k + 1} \right)$$

Hence,

$$\begin{aligned} \frac{9k + 3}{k + 1} &= 5 \\ 9k + 3 &= 5k + 5 \\ 4k &= 2 \\ k &= \frac{2}{4} \\ &= \frac{1}{2} \end{aligned}$$

Thus, point Q divides PR in the ratio 1:2.

Question 3:

Find the ratio in which the YZ-plane divides the line segment formed by joining the points $(-2, 4, 7)$ and $(3, -5, 8)$.

Solution:

Let the YZ plane divide the line segment joining points $(-2, 4, 7)$ and $(3, -5, 8)$ in the ratio $k:1$.

Hence, by section formula, the coordinates of point of intersection are given by

$$\left(\frac{k(3) - 2}{k + 1}, \frac{k(-5) + 4}{k + 1}, \frac{k(8) + 7}{k + 1} \right)$$

On the YZ plane, the x -coordinate of any point is zero.

$$\begin{aligned}\Rightarrow \frac{3k-2}{k+1} &= 0 \\ \Rightarrow 3k-2 &= 0 \\ \Rightarrow k &= \frac{2}{3}\end{aligned}$$

Thus, the YZ-plane divides the line segment formed by joining the given points in the ratio 2 : 1 .

Question 4:

Using section formula, show that the points $A(2, -3, 4)$, $B(-1, 2, 1)$ and $C\left(0, \frac{1}{3}, 2\right)$ are collinear.

Solution:

Let P be a point that divides AB in the ratio $k : 1$.

Hence, by section formula, the coordinates of point of intersection are given by

$$\left(\frac{k(-1)+2}{k+1}, \frac{k(2)-3}{k+1}, \frac{k(1)+4}{k+1} \right)$$

Now, we find the value of k at which point P coincides with point C.

By taking $\frac{-k+2}{k+1} = 0$, we obtain $k = 2$.

For $k = 2$, the coordinates of point P are $\left(0, \frac{1}{3}, 2\right)$

i.e., $\left(0, \frac{1}{3}, 2\right)$ is a point that divides AB externally in the ratio 2 : 1 and is the same as point C.

Hence, points A, B, and C are collinear.

Question 5:

Find the coordinates of the points which trisect the line segment joining the points $P(4, 2, -6)$ and $Q(10, -16, 6)$.

Solution:

Let A and B be the points that trisect the line segment joining the points $P(4, 2, -6)$ and $Q(10, -16, 6)$.

Point A divides PQ in the ratio 1 : 2.

Therefore, by section formula, the coordinates of point A are given by

$$\left(\frac{1(10)+2(4)}{1+2}, \frac{1(-16)+2(2)}{1+2}, \frac{1(6)+2(-6)}{1+2} \right) \\ \Rightarrow (6, -4, -2)$$

Point B divides PQ in the ratio 2:1.

Therefore, by section formula, the coordinates of point B are given by

$$\left(\frac{2(10)+1(4)}{1+2}, \frac{2(-16)+1(2)}{1+2}, \frac{2(6)+1(-6)}{1+2} \right) \\ \Rightarrow (8, -10, 2)$$

Thus, $(6, -4, -2)$ and $(8, -10, 2)$ are the points that trisect the line segment joining the points $P(4, 2, -6)$ and $Q(10, -16, 6)$.

MISCELLANEOUS EXERCISE

Question 1:

Three vertices of a parallelogram ABCD are $A(3, -1, 2)$, $B(1, 2, -4)$ and $C(-1, 1, 2)$. Find the coordinates of the fourth vertex.

Solution:

The three vertices of a parallelogram ABCD are given as $A(3, -1, 2)$, $B(1, 2, -4)$ and $C(-1, 1, 2)$.
Let the coordinates of the fourth vertex be $D(x, y, z)$.

We know that the diagonals of a parallelogram bisect each other.
Therefore, in parallelogram ABCD, diagonals AC and BD bisect each other.

i.e., Mid-point of AC = Mid-point of BD

$$\Rightarrow \left(\frac{3-1}{2}, \frac{-1+1}{2}, \frac{2+2}{2} \right) = \left(\frac{x+1}{2}, \frac{y+2}{2}, \frac{z-4}{2} \right)$$
$$\Rightarrow (1, 0, 2) = \left(\frac{x+1}{2}, \frac{y+2}{2}, \frac{z-4}{2} \right)$$

Hence,

$$\frac{x+1}{2} = 1, \frac{y+2}{2} = 0 \text{ and } \frac{z-4}{2} = 2$$
$$\Rightarrow x = 1, y = -2 \text{ and } z = 8$$

Thus, the coordinates of the fourth vertex D are $(1, -2, 8)$.

Question 2:

Find the lengths of the medians of the triangle with vertices $A(0, 0, 6)$, $B(0, 4, 0)$ and $C(6, 0, 0)$.

Solution:

Let AD, BE and CF be the medians of the given triangle.

Since, AD is the median, D is the mid-point of BC

Coordinates of point $D = \left(\frac{0+6}{2}, \frac{4+0}{2}, \frac{0+0}{2} \right) = (3, 2, 0)$

$$\begin{aligned}
 AD &= \sqrt{(0-3)^2 + (0-2)^2 + (6-0)^2} \\
 &= \sqrt{9+4+36} \\
 &= \sqrt{49} \\
 &= 7
 \end{aligned}$$

Since, BE is the median, E is the mid-point of AC

Coordinates of point $E = \left(\frac{0+6}{2}, \frac{0+0}{2}, \frac{0+6}{2} \right) = (3, 0, 3)$

$$\begin{aligned}
 BE &= \sqrt{(3-0)^2 + (0-4)^2 + (3-0)^2} \\
 &= \sqrt{9+16+9} \\
 &= \sqrt{34}
 \end{aligned}$$

Since CF is the median, F is the mid-point of AB

Coordinates of point $F = \left(\frac{0+0}{2}, \frac{0+4}{2}, \frac{0+6}{2} \right) = (0, 2, 3)$

$$\begin{aligned}
 CF &= \sqrt{(6-0)^2 + (0-2)^2 + (0-3)^2} \\
 &= \sqrt{36+4+9} \\
 &= \sqrt{49} \\
 &= 7
 \end{aligned}$$

Thus, the lengths of the medians of triangle ABC are $7, \sqrt{34}$ and 7 .

Question 3:

If the origin is the centroid of the triangle PQR with vertices $P(2a, 2, 6)$, $Q(-4, 3b, -10)$ and $R(8, 14, 2c)$, then find the values of a, b and c .

Solution:

It is known that the coordinates of the centroid of the triangle, whose vertices are

$$(x_1, y_1, z_1), (x_2, y_2, z_2) \text{ and } (x_3, y_3, z_3) \text{ are } \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3} \right)$$

Therefore, coordinates of the centroid of

$$\begin{aligned}
 \Delta PQR &= \left(\frac{2a-4+8}{3}, \frac{2+3b+14}{3}, \frac{6-10+2c}{3} \right) \\
 &= \left(\frac{2a+4}{3}, \frac{3b+16}{3}, \frac{2c-4}{3} \right)
 \end{aligned}$$

It is given that origin $(0,0,0)$ is the centroid of the ΔPQR
Hence,

$$\frac{2a+4}{3}=0, \frac{3b+16}{3} \text{ and } \frac{2c-4}{3}=0$$

Thus, the values of $a=-2, b=-\frac{16}{3}$ and $c=2$.

Question 4:

Find the coordinates of a point on y -axis which are at distance of $5\sqrt{2}$ from the point $P(3,-2,5)$.

Solution:

If a point is on the y -axis, then x -coordinate and the z -coordinate of the point are zero.

Let $A(0,b,0)$ be the point on the y -axis at a distance of $5\sqrt{2}$ from point $P(3,-2,5)$.

Accordingly, $AP=5\sqrt{2}$

On squaring both sides, we obtain $AP^2=50$

Therefore,

$$(3-0)^2 + (-2-b)^2 + (5-0)^2 = 50$$

$$9 + 4 + b^2 + 4b + 25 = 50$$

$$b^2 + 4b - 12 = 0$$

$$b^2 + 6b - 2b - 12 = 0$$

$$(b+6)(b-2) = 0$$

$$\Rightarrow b = -6 \text{ and } b = 2$$

Thus, the coordinates of the required point are $(0,2,0)$ and $(0,-6,0)$.

Question 5:

A point R with x -coordinate 4 lies on the line segment joining the points $P(2,-3,4)$ and $Q(8,0,10)$. Find the coordinates of the point R.

[Hint Suppose R divides PQ in the ratio $k:1$. The coordinates of the point R are given

$$\text{by } \left(\frac{8k+2}{k+1}, \frac{-3}{k+1}, \frac{10k+4}{k+1} \right)]$$

Solution:

The coordinates of points P and Q are given as $P(2, -3, 4)$ and $Q(8, 0, 10)$.

Let R divide the line segment PQ in the ratio $k:1$.

Hence, by section formula, the coordinates of point R are given by

$$\left(\frac{k(8)+2}{k+1}, \frac{k(0)-3}{k+1}, \frac{k(10)+4}{k+1} \right) \\ \Rightarrow \left(\frac{8k+2}{k+1}, \frac{-3}{k+1}, \frac{10k+4}{k+1} \right)$$

It is given that the x -coordinate of point R is 4.

Hence,

$$\frac{8k+2}{k+1} = 4 \\ 8k+2 = 4k+4 \\ 4k = 2 \\ k = \frac{1}{2}$$

Therefore, the coordinates of point R are

$$\left(4, \frac{-3}{\frac{1}{2}+1}, \frac{10\left(\frac{1}{2}\right)+4}{\frac{1}{2}+1} \right) \\ \Rightarrow (4, -2, 6)$$

Thus, the coordinates of point R are $(4, -2, 6)$.

Question 6:

If A and B be the points $(3, 4, 5)$ and $(-1, 3, -7)$ respectively, find the equation of the set of points P such that $PA^2 + PB^2 = k^2$, where k is a constant.

Solution:

The coordinates of points A and B are given as $A(3, 4, 5)$ and $B(-1, 3, -7)$ respectively.

Let the coordinates of point P be (x, y, z) .

On using distance formula, we obtain

$$\begin{aligned}
 PA^2 &= (x-3)^2 + (y-4)^2 + (z-5)^2 \\
 &= x^2 + 9 - 6x + y^2 + 16 - 8y + z^2 + 25 - 10z \\
 &= x^2 - 6x + y^2 - 8y + z^2 - 10z + 50
 \end{aligned}$$

$$\begin{aligned}
 PB^2 &= (x+1)^2 + (y-3)^2 + (z+7)^2 \\
 &= x^2 + 1 + 2x + y^2 + 9 - 6y + z^2 + 49 + 14z \\
 &= x^2 + 2x + y^2 - 6y + z^2 + 14z + 59
 \end{aligned}$$

Now, if $PA^2 + PB^2 = k^2$, then,

$$\begin{aligned}
 &\Rightarrow (x^2 - 6x + y^2 - 8y + z^2 - 10z + 50) + (x^2 + 2x + y^2 - 6y + z^2 + 14z + 59) = k^2 \\
 &\Rightarrow 2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z + 109 = k^2 \\
 &\Rightarrow 2(x^2 + y^2 + z^2 - 2x - 7y + 2z) = k^2 - 109 \\
 &\Rightarrow x^2 + y^2 + z^2 - 2x - 7y + 2z = \frac{k^2 - 109}{2}
 \end{aligned}$$

Thus, the required equation is $x^2 + y^2 + z^2 - 2x - 7y + 2z = \frac{k^2 - 109}{2}$.